

The light U boson as the mediator of a new force, coupled to a combination of Q , B , L and dark matter

Pierre FAYET

*Laboratoire de Physique Théorique de l'École Normale Supérieure
24 rue Lhomond, 75231 Paris cedex 05, France **

and Centre de Physique Théorique, École polytechnique, 91128 Palaiseau cedex, France †

A new light gauge boson U may have both vector and axial couplings. In a large class of theories however, the new $U(1)$ current J_F^μ naturally combines with the weak neutral current $J_{Z_{\text{SM}}}^\mu$, both parity-violating, into a vectorial current J_U^μ , combination of the B , L and electromagnetic currents with a possible dark matter current.

U^μ may be expressed equivalently as $\cos \xi C^\mu + \sin \xi Z_{\text{SM}}^\mu$ (“mixing with the Z ”) or $(1/\cos \chi) \hat{C}^\mu + \tan \chi A^\mu$ (“mixing with the photon”), with \hat{C} coupled to B , L and dark matter. The U boson may be viewed as a generalized dark photon, coupled to SM particles through $Q_U = Q + \lambda_B B + \lambda_i L_i$, with strength $g'' \cos \xi \cos^2 \theta = e \tan \chi$. “Kinetic mixing” terms, gauge invariant or not, simply correspond to a description in a non-orthogonal field basis (rather than to a new physical effect), with the dark photon in general also coupled to B and L .

In a grand-unified theory $Q_U^{\text{gut}} = Q - 2(B - L)$ at the GUT scale for SM particles, invariant under the $SU(4)$ electrostrong symmetry group, with a non-vanishing $\epsilon = \tan \chi$ already present at the GUT scale, leading to $Q_U \simeq Q - 1.64(B - L)$ at low energy. This also applies, for a very light or massless U boson, to a new long-range force, which could show up through apparent violations of the Equivalence Principle.

Keywords: U boson, dark photon, new interaction, Equivalence Principle tests, grand-unification

Preprint LPTENS 16/07 November 15, 2016

I. INTRODUCTION

The possible existence of a light neutral spin-1 boson with a small gauge coupling, in the \sim MeV to hundred MeVs mass range and decaying most notably into e^+e^- pairs, has been studied for a long time [1–3]. It is generally expected not to have a significant effect on neutral current phenomenology at higher q^2 , as compared to a heavy Z' , but it could affect the anomalous magnetic moments of the muon or electron, parity-violation effects in atomic physics, or be produced in various decays and beam dump experiments, etc..

In an electroweak theory the new boson, referred to as the U boson, can mix with the Z and with the photon through a 3×3 matrix, borrowing features from both kinds of particles. The Z weak neutral current as well as the electromagnetic current, and also the B and L currents, can contribute to the new current J_U^μ , which has in general both vector and axial parts. This could

however lead to too-strong parity-violation effects [4], with axial couplings also enhancing the cross-sections for longitudinally-polarized U bosons, produced much like light pseudoscalars [1].

In a large class of spontaneously broken gauge theories however, the U couplings to quarks and leptons are *naturally vectorial in the small mass limit*. The U current, obtained from a mixing of the extra- $U(1)$ current J_F^μ with the Z current, ultimately involves a combination of the baryonic, leptonic (or $B - L$) and electromagnetic currents [5]. This includes and generalizes, within the framework of an extended electroweak or grand-unified theory, the very specific case of a “dark photon” coupled to electric charges, which has focused much of the experimental attention. This also applies to an extremely light or massless U boson inducing a new long-range force, extremely weak, which could lead to apparent deviations from the Equivalence Principle [3, 5, 6].

The U boson may also couple to dark matter, and can mediate sufficient annihilations through *stronger-than-weak* interactions so as to allow for dark matter particles to be light [7, 8]. It may provide a possible explanation for the apparent discrepancy between the expected and measured values of $g_\mu - 2$ [9–11], and be at the origin of many interesting effects.

* CNRS UMR 8549, ENS, PSL Research University & UPMC, Sorbonne Universités

† CNRS UMR 7644, Université Paris-Saclay

II. A VECTORIALLY-COUPLED U BOSON

In the presence of weak-interactions one may expect the U couplings to be parity-violating. Still this one may be vectorially coupled to quarks and leptons, as in the specific case of a “dark photon”, but allowing for more general situations. This occurs in a large class of models in which the extended electroweak symmetry is broken by a single doublet; or by two (or more) as in supersymmetric theories, but with the same gauge quantum numbers. The symmetry breaking may then be viewed as induced by a single active doublet φ , the others being “inert”.

A. Q_U as a combination of Q , B , L and F_{dark}

We express the extended electroweak covariant derivative as

$$iD_\mu = i\partial_\mu - \underbrace{g\mathbf{T}\cdot\mathbf{V}_\mu - \frac{g'}{2}YB_\mu - \frac{g''}{2}FC_\mu}_{\text{(possibly grand-unified)}}, \quad (1)$$

with the extra $U(1)_F$ gauge symmetry commuting with $SU(3)_C \times SU(2) \times U(1)_Y$, or $SU(5)$ in a grand-unified theory. The Lagrangian density, expressed as usual in an orthonormal field basis, includes the couplings of the gauge fields with the corresponding currents,

$$\mathcal{L} = -\frac{1}{4}(\mathbf{V}^{\mu\nu}\mathbf{V}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu} + C^{\mu\nu}C_{\mu\nu}) - \mathcal{J}^\mu \cdot \mathbf{V}_\mu - \mathcal{J}_Y^\mu B_\mu - \mathcal{J}_F^\mu C_\mu + \dots \quad (2)$$

When quarks and leptons acquire their masses from a single electroweak doublet as in the standard model, or several but with the same gauge quantum numbers, the gauge invariance of their Yukawa couplings requires the new $U(1)_F$ quantum number F , and the corresponding current \mathcal{J}_F^μ , to be expressed as [5]

$$F = \alpha_B B + \beta_i L_i + \gamma Y + F_d, \quad (3)$$

$$\mathcal{J}_F^\mu = \frac{g''}{2} J_F^\mu = \alpha_B \mathcal{J}_B^\mu + \beta_i \mathcal{J}_{L_i}^\mu + \gamma \mathcal{J}_Y^\mu + \mathcal{J}_d^\mu.$$

The new $U(1)$ current is naturally expressed as a linear combination of the B and L currents with the weak hypercharge current [40], and a possible dark-matter or extra spin-0 singlet contribution associated with a “hidden sector”. The simultaneous appearance of B , L and Y in (3) is actually required in the framework of grand-unification, to ensure that $U(1)_F$ commutes with the non-abelian grand-unification gauge group [3, 5].

For $\gamma=0$ the new interaction, coupled to a linear combination of the baryon and lepton numbers, is simple to study in terms of the mass and couplings of the new boson, then unmixed with the Z and the photon. For the theory to be anomaly-free the U current may be taken as the $B-L$ current (in the presence of ν_R fields), possibly combined with the $L_i - L_j$ and dark matter currents.

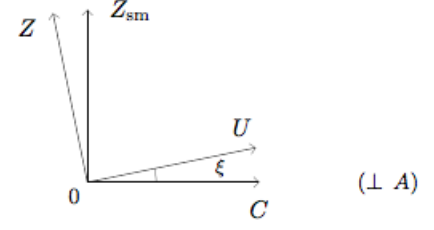


FIG. 1: “Mixing with the Z ”, in the plane orthogonal to the photon field A^μ . U^μ is a (small) mixing of C^μ (coupled to $F = Y + \alpha_B B + \beta_i L_i + F_d$) with Z_{sm}^μ , leading to \mathcal{J}_U^μ as in (7,8), with $\tan \xi = g''/\sqrt{g^2 + g'^2}$.

The new force may be of infinite or finite range, and may also act on dark matter particles.

We now concentrate on the more elaborate situation of a $U(1)_F$ gauge interaction of C^μ in (1-3) involving the weak hypercharge generator Y , allowing to normalize g'' and F so that $\gamma = 1$. We may again use $B-L$ and L_i-L_j in expression (3) of F for the theory to be anomaly-free, including ν_R fields. The v.e.v. $v/\sqrt{2} \simeq 174$ GeV of the doublet φ , with $Y = 1$, breaks the $SU(3)_C \times SU(2) \times U(1)_Y \times U(1)_F$ gauge symmetry to $SU(3)_C \times U(1)_{\text{QED}} \times U(1)_U$, with $m_W = gv/2$. The three neutral fields W_3 , B and C are mixed into the massless photon field A , the massive Z field and a new neutral field U , still massless at this stage. They are given by [5]

$$\begin{cases} A = \frac{g'W_3 + gB}{\sqrt{g^2 + g'^2}} = \sin \theta W_3 + \cos \theta B, \\ Z = \frac{gW_3 - g'B - g''C}{\sqrt{g^2 + g'^2 + g''^2}}, \\ U = \frac{g''(gW_3 - g'B) + (g^2 + g'^2)C}{\sqrt{g^2 + g'^2} \sqrt{g^2 + g'^2 + g''^2}}. \end{cases} \quad (4)$$

The photon field has its usual SM expression in terms of $\tan \theta = g'/g$. Z and U are obtained by rotating the standard $Z_{\text{sm}} = \cos \theta W_3 - \sin \theta B$ and the $U(1)_F$ gauge field C in the plane orthogonal to A as represented in Fig. 1, according to

$$\begin{cases} Z = \cos \xi Z_{\text{sm}} - \sin \xi C, \\ U = \sin \xi Z_{\text{sm}} + \cos \xi C, \end{cases} \quad (5)$$

with

$$\tan \xi = \frac{g''}{\sqrt{g^2 + g'^2}} = \frac{g''}{gZ}. \quad (6)$$

This leads to the 3×3 orthogonal mixing matrix in (4).

The U field is still massless at this stage, and coupled to a conserved current. This one is obtained, in the small m_U limit in which U is almost exactly given by (4), as

a combination of the $U(1)_F$ current J_F^μ with the standard weak neutral current $J_{Z_{\text{sm}}}^\mu$, using $\sin \xi \sqrt{g^2 + g'^2} = g'' \cos \xi$ from (6). Including as in (2,3) the coupling constants $g, g'/2, g''/2$ within the currents \mathcal{J}^μ we have

$$\begin{aligned} \mathcal{J}_U^\mu &= \cos \xi \left(\underbrace{\frac{g''}{2} J_F^\mu}_{\mathcal{J}_F^\mu} + \sin \xi \underbrace{\sqrt{g^2 + g'^2} (J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu)}_{\mathcal{J}_{Z_{\text{sm}}}^\mu} \right) \\ &= g'' \cos \xi \left[\frac{1}{2} J_Y^\mu + \frac{1}{2} (\alpha_B J_B^\mu + \beta_i J_{L_i}^\mu + J_d^\mu) \right. \\ &\quad \left. + (J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu) \right]. \end{aligned} \quad (7)$$

With $Y/2$ (from J_F^μ) and T_3 (from $J_{Z_{\text{sm}}}^\mu$) combining into $Q = \frac{Y}{2} + T_3$ the axial part in the U current for quarks and leptons disappears in the massless U limit. The resulting current is

$$\begin{aligned} \mathcal{J}_U^\mu &= g'' \cos \xi \left(\cos^2 \theta J_{\text{em}}^\mu + \frac{1}{2} (\alpha_B J_B^\mu + \beta_i J_{L_i}^\mu + J_d^\mu) \right) \\ &= e \tan \chi \left(J_{\text{em}}^\mu + \frac{1}{2 \cos^2 \theta} (\alpha_B J_B^\mu + \beta_i J_{L_i}^\mu + J_d^\mu) \right). \end{aligned} \quad (8)$$

It is associated with a conserved charge [5], normalized as

$$Q_U = Q + \frac{1}{2 \cos^2 \theta} (\alpha_B B + \beta_i L_i + F_d), \quad (9)$$

with coupling constant

$$\begin{aligned} g'' \cos \xi \cos^2 \theta &= g'' \sqrt{\frac{g^2 + g'^2}{g^2 + g'^2 + g''^2}} \frac{g^2}{g^2 + g'^2} \\ &= \epsilon e = e \tan \chi \simeq g'' \cos^2 \theta. \end{aligned} \quad (10)$$

This coupling is expressed in terms of the elementary charge $e = gg'/\sqrt{g^2 + g'^2}$ as ϵe with $\epsilon = \tan \chi$. The angle χ may be interpreted from another expression of U^μ , given in a non-orthogonal basis as $(1/\cos \chi) \hat{C}^\mu + \tan \chi A^\mu$ (cf. eqs. (46,48,55) and Fig. 3 in Sec. V). \hat{C}^μ is in general coupled, not only to dark matter and to a spin-0 field σ in the hidden sector responsible for m_U [5], but also to B and L as well. \hat{C}^μ and A^μ , both orthogonal to Z^μ in (4), are themselves non-orthogonal, at an angle $\frac{\pi}{2} + \chi$ with $\tan \chi = \epsilon$ given by (10), very close to $\frac{\pi}{2}$ if the extra $U(1)$ coupling g'' is small.

B. Relations between U charges

The U charges in (9), expressed for the first generation of quarks and leptons as a linear combination of Q, B and L_e , satisfy the same additivity relations

$$\begin{aligned} Q_U(p) - Q_U(n) &= Q_U(u) - Q_U(d) \\ &= Q_U(\nu_e) - Q_U(e) = Q_U(W^+) = 1. \end{aligned} \quad (11)$$

They express the conservation of Q_U in the limit of a massless U , associated with an unbroken symmetry $U(1)_U$, with the W^\pm carrying ± 1 unit of Q_U . The corresponding vector couplings of the U boson, expressed from (8-10) as

$$\begin{cases} f_{\nu_e} = \epsilon e Q_U(\nu_e), & f_p = 2f_u + f_d = \epsilon e Q_U(p), \\ f_e = \epsilon e Q_U(e), & f_n = f_u + 2f_d = \epsilon e Q_U(n), \end{cases} \quad (12)$$

must therefore verify

$$f_n = f_p + f_e - f_{\nu_e}. \quad (13)$$

This relation expresses in particular that Q_U remains conserved in the β decay of the neutron, $n \rightarrow p e \bar{\nu}_e$.

The U is constrained to interact very weakly with electrons, so that the extra contribution $\delta_U a_e$ to the electron anomaly be sufficiently small; and with protons, so that the $\pi^0 \rightarrow \gamma U$ decay amplitude, proportional to $f_p = 2f_u + f_d$, be sufficiently small (cf. Sec. VI). It should also interact sufficiently weakly with neutrinos so as to satisfy the constraint $|f_{\nu_e} f_e|/m_U^2 \lesssim G_F$ from low- q^2 ν_e - e scattering, i.e. [4, 7]

$$|f_{\nu_e} f_e|^{1/2} \lesssim 3 \cdot 10^{-6} m_U (\text{MeV}), \quad (14)$$

valid for m_U larger than a few MeVs. The additivity property (11-13) implies that if f_e, f_p and f_{ν_e} are all small, the coupling to the neutron $f_n = f_p + f_e - f_{\nu_e}$ is expected to be small as well.

The $U(1)_F$ generator in (3) may well involve B and L through their difference $B - L$, in view of an anomaly-free theory (including ν_R 's), or of grand-unification as we shall see. Q_U may then be expressed as

$$Q_U = Q - \lambda (B - L) + Q_{U\text{dark}}. \quad (15)$$

More specifically with $\lambda \simeq 1$,

$$Q_U \simeq Q - (B - L) + Q_{U\text{dark}}, \quad (16)$$

would lead to smaller interactions with the proton and the electron, i.e.

$$\text{small } (f_p = -f_e), \text{ as compared to } (f_n = -f_{\nu_e}), \quad (17)$$

very much as found in [3], in the presence of axial couplings. In a similar way Q_U close to $Q - B + Q_{Ud}$ (or $Q - (B - 3L_\tau) + Q_{Ud}$ in view of an anomaly-free theory) would lead to small f_p and f_{ν_e} , with a larger $f_n \simeq f_e$, again in agreement with (13).

Such relations, however, may be avoided in other situations, with two spin-0 doublets at least, in which the U current is not naturally vectorial and conserved in the small m_U limit, so that significant parity-violation effects may have to be expected. A light U in a longitudinal polarisation state may then be produced and interact significantly, much like the spin-0 pseudoscalar a associated with the spontaneous breaking of the global $U(1)_U$ [1]. Both effects restrict significantly the possible size of axial couplings (cf. subsection VI A).

C. A new long-range force, and Equivalence Principle tests

The U could stay massless, mediating a new long-range force acting *additively* on ordinary particles, proportionally to a linear combination of B (as considered long ago by Lee and Yang [12]), with L and Q . Or it may acquire a mass if the $U(1)_U$ symmetry gets spontaneously broken. The fact that both B and L can be present simultaneously in the expression of Q_U , and in combination with the electric charge Q , allows for an extension to grand-unified theories. The fact that Q may also appear alone illustrates that the popular dark photon case is included as a specific case of this general analysis.

For ordinary matter Q_U appears as a combination of the numbers of protons, neutrons and electrons, or Z , N and Q , i.e. effectively Z and N only for ordinary neutral matter [6]. More specifically the new force may act mostly on the number of neutrons N , as in the case of a “protophobic” U boson for which Q_U is close to $Q - (B - L)$ [3].

With $Q_U = Q - \lambda(B - L) + Q_{Ud}$ as in (15) (and also in the absence of the Q term, if Y does not appear in expression (3) of F so that the U does not mix with the Z and the photon), one has

$$Q_U(p + e) = 0, \quad Q_U(n) = -\lambda, \quad (18)$$

so that

$$Q_U = -\lambda N, \quad (19)$$

for ordinary neutral matter. The interaction potential between two bodies of mass m_i and number of neutrons N_i is then given by

$$V(r) = -\frac{G_N m_1 m_2}{r} + \frac{(\lambda e)^2 e^2 N_1 N_2}{4\pi\epsilon_0 r} e^{-r/\frac{\hbar}{m_U c}}. \quad (20)$$

The ratio between the repulsive U -exchange potential and the gravitational potential between two neutrons at a distance r somewhat larger than $\hbar/m_U c$ is about

$$\frac{V_U(r)}{V_g(r)} \simeq -(\lambda e)^2 \alpha \left(\frac{m_{\text{Planck}}}{m_n} \right)^2 \simeq -1.23 \cdot 10^{36} (\lambda e)^2. \quad (21)$$

For a massless or almost massless U boson the new force could lead to apparent deviations from the Equivalence Principle [5, 6], constraining it to be considerably weaker than gravitation, by $\approx 10^{-10}$ at least, corresponding typically to

$$\lambda e \lesssim 10^{-23}, \quad (22)$$

depending also on m_U and $\lambda_U = \hbar/m_U c$, so that the resulting violations of the Equivalence Principle be $\lesssim 10^{-13}$ [13–16].

The MICROSCOPE experiment will soon test the validity of this principle at the 10^{-15} level [17]. The additivity property of the new force induced by a spin-1 U boson, following from the linear expression (9) of Q_U (as opposed to an hypothetical coupling to mass, or strangeness, ...), is also in contrast with the case of a spin-0 mediator [18], for which other contributions to the expression of the new force are generally expected. This may allow for a *distinction between spin-1 and spin-0 mediators*, should a deviation from the Equivalence Principle be observed.

D. Generating a small mass for the U boson

The U boson can acquire a small mass from a neutral singlet σ with $Y = 0$, directly providing $A = \sin\theta W_3 + \cos\theta B$ in (4) as the massless photon field [5]. The singlet v.e.v. $\langle \sigma \rangle = w/\sqrt{2}$ generates a mass term $m_C = g'' F_\sigma w/2$, resulting in a small U mass

$$m_U \simeq m_C \cos\xi \simeq g'' F_\sigma w/2, \quad (23)$$

with $\sqrt{2} \Re\sigma$ a physical singlet BEH field, possibly (slightly) mixed with the standard one $\sqrt{2} \Re\varphi^0$ taken to describe the new 125 GeV spin-0 boson [19, 20].

The massive U field differs very little from its expression in (4,5), through a tiny change in ξ (from ξ_0 to $\xi_0 + \delta\xi$) inducing very small parity-violating contributions $\approx m_U^2/m_Z^2$ in the U current J_U^μ . To discuss these small m_U corrections to the U and Z currents we observe that the theory is invariant under a simultaneous change of sign for C^μ and g'' , acting as

$$C^\mu \rightarrow -C^\mu, \quad g'' \rightarrow -g'', \quad (24)$$

so that

$$(A, Z, U) \rightarrow (A, Z, -U), \quad (25)$$

$$\text{with } (g'', \xi, \chi, m_U) \rightarrow -(g'', \xi, \chi, m_U).$$

Corrections to J_U^μ in (7,8), odd in g'' and vanishing with m_U , are thus $\approx g'' \cos\xi m_U^2/m_Z^2$ (rather than $g'' \cos\xi m_U/m_Z$), i.e.

$$\approx e \tan\chi m_U^2/m_W^2. \quad (26)$$

The Z current differs also very little from its standard SM expression with a small contribution from J_F^μ , by terms $\approx m_U^2/m_Z^2$ as obtained from (4-8).

E. Special case of the dark photon

The special case for which B and L do not participate in expression (3) of the $U(1)_F$ quantum number, simply reduced to

$$F = Y + F_d, \quad (27)$$

provides in an electroweak theory a “dark photon”, with the U coupled to standard model particles proportionally to their electromagnetic current, through

$$\mathcal{J}_U^\mu = \underbrace{g'' \cos \xi \cos^2 \theta}_{\epsilon e} (J_{\text{em}}^\mu + \frac{1}{2 \cos^2 \theta} J_d^\mu). \quad (28)$$

The coupling $g'' \cos \xi \cos^2 \theta$ may be expressed as in (10) as ϵe in terms of the elementary charge $e = gg'/\sqrt{g^2 + g'^2}$, with

$$\epsilon = \tan \chi = \frac{g''}{g'} \frac{g}{\sqrt{g^2 + g'^2 + g''^2}} \simeq \frac{g''}{g'} \cos \theta, \quad (29)$$

in the small m_U limit.

This simple situation, a special case of the general one, has been obtained in an extended electroweak theory without ever referring to a largely fictitious “kinetic mixing” term. Such terms are simply associated with a description in a *non-orthogonal field basis, independently of the fact that they are gauge invariant or not*. Adding them explicitly in an initial Lagrangian density (then under the restrictive condition that they must be gauge invariant [21, 22]) does not provide additional physical freedom. Indeed the notion of scalar product is not generalized by adding to its usual expression $xx' + yy'$ in orthonormal coordinates a non-diagonal “mixing term” $\epsilon(xy' + yx')$. Discussing a theory in a non-orthogonal rather than in an orthonormal field basis has no effect on the results.

The simple situation of a “dark photon” has been the focus of much experimental attention recently [23, 24]. But it appears excessively restrictive as compared to the general situation for a U boson [5], excluding possible contributions from the B and L currents, that ought to be present in a grand-unified theory.

III. THE U CURRENT IN GRAND-UNIFIED THEORIES

Indeed within $SU(5)$ -type grand-unified theories [25, 26], the weak-hypercharge Y , now a generator of $SU(5)$, is no longer abelian, while the $U(1)_F$ generator F in (1,3) should commute with $SU(5)$. It may then look like gauge invariance prevents Y from entering in the expression of the $U(1)_F$ quantum number F for the visible particles, requiring $\epsilon = \tan \chi$ to vanish at tree-level, as commonly believed [24].

This is not true however, as Y can contribute to F through the $SU(5)$ -invariant combination involving $B - L$ [5],

$$\begin{aligned} F &= Y - \frac{5}{2} (B - L) \\ &= \begin{cases} -\frac{1}{2} : \mathbf{10} & \text{of } SU(5) \quad \{u_L, d_L, \bar{u}_L, e_L^+\}, \\ \frac{3}{2} : \mathbf{\bar{5}} & \text{of } SU(5) \quad \{\bar{d}_L, \nu_L, e_L^-\}, \\ 1 : \mathbf{\bar{5}}_H & \text{of } SU(5) \quad \text{including } \varphi. \end{cases} \quad (30) \end{aligned}$$

One also has $F = -\frac{5}{2}$ for possible $\bar{\nu}_L$ singlets describing right-handed neutrinos ν_R , making the theory anomaly-free, and the $U(1)_F$ generator traceless. All Yukawa couplings proportional to $\mathbf{\bar{5}}_H \cdot \mathbf{\bar{5}} \cdot \mathbf{10}$ and $\mathbf{\bar{5}}_H \cdot \mathbf{10} \cdot \mathbf{10}$, responsible for down-quarks and charged-lepton masses and up-quark masses, respectively, have $F = 0$ and are invariant under $SU(5) \times U(1)_F$.

The $U(1)_F$ current, including its hidden-sector part J_d^μ , now reads

$$J_F^\mu = J_Y^\mu - \frac{5}{2} J_{B-L}^\mu + J_d^\mu. \quad (31)$$

By combining it with $J_{Z_{\text{sm}}}^\mu$ as in (7,8) we get the U current

$$\begin{aligned} J_U^\mu &= g'' \cos \xi \left(\frac{1}{2} J_Y^\mu - \frac{5}{4} J_{B-L}^\mu + \frac{1}{2} J_d^\mu + (J_3^\mu - \sin^2 \theta J_{\text{em}}^\mu) \right) \\ &= g'' \cos \xi \left(\cos^2 \theta J_{\text{em}}^\mu - \frac{5}{4} J_{B-L}^\mu + \frac{1}{2} J_d^\mu \right), \quad (32) \end{aligned}$$

associated with the charge [5]

$$Q_U = Q - \frac{5}{4 \cos^2 \theta} (B - L) + \frac{1}{2 \cos^2 \theta} F_d. \quad (33)$$

The corresponding current J_U^μ involves a term proportional to the electromagnetic current J_{em}^μ , with the same coupling

$$g'' \cos \xi \cos^2 \theta = \epsilon e = e \tan \chi \quad (34)$$

as in (10,29), and a term proportional to the $B - L$ current, plus a dark matter current.

IV. Q_U , THE ELECTROSTRONG SYMMETRY, AND $B - L$

A. Q_U at the grand-unification scale

To understand better the origin and meaning of the new charge we note that Q_U in (33), if evaluated at the grand-unification scale with $g' = g\sqrt{3/5}$ and $\sin^2 \theta = 3/8$, would read

$$\text{at GUT scale: } Q_U^{\text{gut}} = Q - 2(B - L) + \frac{4}{5} F_d. \quad (35)$$

This Q_U^{gut} is invariant under the $SU(4)_{\text{es}} (\sim O(6))$ *electrostrong* symmetry group unifying directly electromagnetism with strong interactions, including $SU(3)_C \times U(1)_{\text{QED}}$ within $SU(5)$, and commuting with $U(1)_U$.

Indeed, forgetting momentarily about the **24** adjoint v.e.v. (or extra dimensions) responsible for the breaking of $SU(5) \times [U(1)_F]$ into $SU(3)_C \times SU(2) \times U(1)_Y \times [U(1)_F]$, the v.e.v. of the quintuplet $\mathbf{5}_H$ (including the electroweak doublet φ , with $F = Y = 1$), is responsible for the symmetry breaking

$$SU(5) \times U(1)_F \xrightarrow{\langle \mathbf{5}_H \rangle} SU(4)_{\text{es}} \times U(1)_U. \quad (36)$$

It leaves at this stage unbroken the $U(1)_U$ group generated by Q_U^{gut} in (35), commuting with $SU(4)_{\text{es}}$. This remaining $U(1)_U$ may then be broken by the singlet v.e.v. $\langle \sigma \rangle = w/\sqrt{2}$, generating a non-vanishing m_U as in (23).

This U charge, invariant at the GUT scale under the $SU(4)_{\text{es}}$ electrostrong symmetry group, is the same for all components within $SU(4)_{\text{es}}$ representations, i.e. for quarks and leptons

$$Q_U^{\text{gut}} = \begin{cases} 1 : \mathbf{\bar{4}} & \begin{pmatrix} \bar{d} \\ e^- \end{pmatrix}_{L+R}, \\ 0 : \mathbf{\bar{6}} & \{u, \bar{u}\}_L, \\ 2 : \mathbf{1} & \nu_L (+\nu_R), \end{cases} \quad (37)$$

with family indices omitted for simplicity. Dirac quark and lepton fields (excepted possibly for chiral neutrinos) are in vectorial representations of $SU(4)_{\text{es}} \times U(1)_U$, with the U boson interactions, as well as electrostrong interactions, invariant under $SU(4)_{\text{es}}$, preserving parity.

For the 24+1 gauge bosons of $SU(5) \times U(1)_F$, which includes $SU(4)_{\text{es}} \times U(1)_U$ as a subgroup, one has

$$Q_U^{\text{gut}} = \begin{cases} 0 : \mathbf{16} & \{\text{gluons}, \gamma, X^{\pm 4/3}\}, \\ 0 : \mathbf{1} & Z, U, \\ 1 : \mathbf{4} & \begin{pmatrix} Y^{-1/3} \\ W^+ \end{pmatrix}, \\ -1 : \mathbf{\bar{4}} & \begin{pmatrix} Y^{1/3} \\ W^- \end{pmatrix}. \end{cases} \quad (38)$$

Q_U^{gut} vanishes as required for the $\{u, \bar{u}\}$ self-conjugate sextets (the U being “ u -phobic” at the GUT scale), with

$$Q_U^{\text{gut}} = Q - 2(B-L) : \begin{cases} Q_U^p = Q_U^d = -1, & Q_U^\nu = 2, \\ Q_U^n = 2Q_U^d = -2, & Q_U^e = 1, \\ Q_U^u = 0. \end{cases} \quad (39)$$

B. Q_U at low energy

Below the grand-unification scale expressions (35, 37-39) of Q_U get modified as in (15) according to

$$Q_U = Q - \lambda(B-L) + Q_{U\text{dark}}, \quad (40)$$

down to (33),

$$Q_U \simeq Q - 1.64(B-L) = \begin{cases} p : -.64; & \nu : 1.64, \\ n : -1.64; & e : 0.64. \end{cases} \quad (41)$$

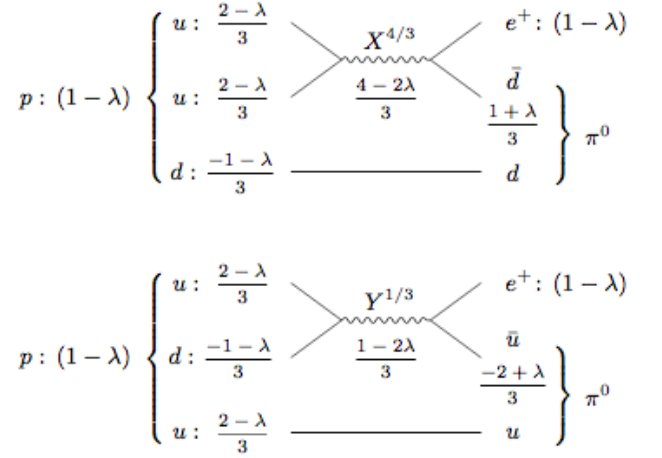


FIG. 2: Two of the diagrams responsible for $p \rightarrow \pi^0 e^+$ in a $SU(5) \times U(1)_F$ theory, showing the values of $Q_U = Q - \lambda(B-L) + Q_{\text{dark}}$ and illustrating its conservation. $\lambda = 2$ at the GUT scale (with $Q_U = 0$ for u and $X^{4/3}$), down to 1.64 at low energy. $Q_U = 1 - \lambda \simeq -.64$ for both the proton and the positron emitted in its decay.

$\lambda = 5/(4 \cos^2 \theta)$ in (40) decreases from 2 at the GUT scale to $\simeq 1.64$ at low energy, with $\sin^2 \theta \simeq .238$.

This is in agreement with the conservation of Q_U (following from those of Q and $B-L$) by all interactions, strong, electroweak and grand-unified, in the massless U limit, with

$$Q_U = \begin{cases} W^\pm : & \pm 1, \\ Y^{\pm 1/3} : & \pm (Q_U^u + Q_U^d) = \pm \frac{1-2\lambda}{3}, \\ X^{\pm 4/3} : & \pm 2Q_U^u = \pm \frac{4-2\lambda}{3}, \end{cases} \quad (42)$$

and

$$Q_U = \begin{cases} u : & \frac{2-\lambda}{3}, & p : 1-\lambda, \\ d : & \frac{-1-\lambda}{3}, & n : -\lambda, \\ \nu : & \lambda, \\ e : & \lambda-1, \end{cases} \quad (43)$$

where $\lambda = 5/(4 \cos^2 \theta) \simeq 1.64$ at low energy. The conservation of Q_U by all interactions, including those induced by $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ exchanges that could be responsible for proton decay, still unobserved [27], is illustrated in Fig. 2.

Note that $\epsilon = \tan \chi \simeq (g''/g') \cos \theta$ is already present at the grand-unification scale. As C^μ is decoupled from the visible sector for $g'' = 0$ (or $\tan \chi = 0$) [41], $\epsilon = \tan \chi$ may be viewed as a measure of the coupling between the visible and hidden sectors. Furthermore a non-vanishing $\epsilon = \tan \chi$ cannot be generated just from quantum corrections once $g'' = 0$ so that ϵ vanishes at tree level, with

C^μ interacting exclusively with the hidden sector, being decoupled from the visible one.

V. THE U AS A DARK PHOTON ALSO COUPLED TO B AND L

A. Another orthogonal basis for neutral gauge fields

The U current has been obtained from the mixing (7,8) between the extra- $U(1)$ current J_F^μ and the standard weak neutral current J_{sm}^μ , providing, in the small m_U limit, a vector current J_U^μ including a contribution $\propto J_{\text{em}}^\mu$, with additional contributions from B , L and dark matter currents.

This result may also be described in a complementary way, by constructing the same current J_U^μ from the electromagnetic current J_{em}^μ combined with the extra current J_d^μ in the hidden sector (as in the specific “dark photon” case), but also in general with the baryonic and leptonic currents J_B^μ and $J_{L_i}^\mu$. Indeed the weak hypercharge current J_Y^μ in (1,3) may be viewed as coupled to B^μ and C^μ through the single hatted combination

$$\hat{B} = \cos \zeta B + \sin \zeta C, \quad \text{with } \tan \zeta = g''/g'. \quad (44)$$

The doublet φ , with $F = Y = 1$, interacts with \hat{B} with the coupling constant $\hat{g}' = \sqrt{g'^2 + g''^2}$. It does not interact with the orthogonal combination \hat{C} , which remains massless at this stage. $\langle \varphi \rangle$ generates a spontaneous breaking of $SU(2) \times U(1)_{\hat{Y}} \rightarrow U(1)_{\text{QED}}$, leaving also \hat{A} massless. With

$$\tan \hat{\theta} = \hat{g}'/g = \sqrt{g'^2 + g''^2}/g \quad (45)$$

this leads to define the orthonormal basis

$$\left\{ \begin{array}{l} \hat{Z} = \cos \hat{\theta} W_3 - \sin \hat{\theta} \hat{B} = \frac{g W_3 - g' B - g'' C}{\sqrt{g^2 + g'^2 + g''^2}} \\ \quad \quad \quad \equiv Z \text{ in (4)}, \\ \hat{A} = \sin \hat{\theta} W_3 + \cos \hat{\theta} \hat{B} = \\ \quad \quad \quad = \frac{(g'^2 + g''^2) W_3 + g g' B + g g'' C}{\sqrt{g'^2 + g''^2} \sqrt{g^2 + g'^2 + g''^2}}, \\ \hat{C} = -\sin \zeta B + \cos \zeta C = \frac{-g'' B + g' C}{\sqrt{g'^2 + g''^2}}, \end{array} \right. \quad (46)$$

with $\hat{\theta}$ defined by

$$\tan \hat{\theta} = \hat{g}'/g = \sqrt{g'^2 + g''^2}/g. \quad (47)$$

We can also relate the two orthonormal basis in (4,46),

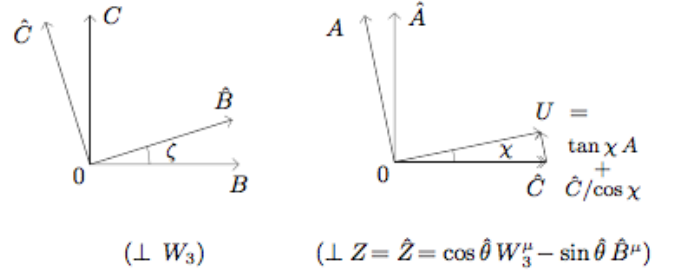


FIG. 3: “Mixing with the photon”, in the plane orthogonal to Z . φ interacts with B^μ and C^μ through $\hat{B}^\mu = \cos \zeta B^\mu + \sin \zeta C^\mu$, leaving $\hat{C}^\mu = -\sin \zeta B^\mu + \cos \zeta C^\mu$ massless. U^μ is a (small) mixing of \hat{C}^μ (coupled to B , L and dark matter) with the photon field A^μ , $U^\mu = (\hat{C}^\mu / \cos \chi) + \tan \chi A^\mu$, leading to $J_U^\mu = (J_{\hat{C}}^\mu / \cos \chi) + e \tan \chi J_{\text{em}}^\mu$ as in (56,59), and earlier in (8).

which have $Z = \hat{Z}$ in common, by writing, for the fields in the orthogonal plane,

$$\begin{cases} \hat{A} = \cos \chi A + \sin \chi U, \\ \hat{C} = -\sin \chi A + \cos \chi U. \end{cases} \quad (48)$$

The angle χ between the \hat{A} field in (46) and the actual photon field A in (4) is obtained from the scalar products

$$\cos \chi = A \cdot \hat{A} = U \cdot \hat{C}, \quad \sin \chi = -A \cdot \hat{C} = \hat{A} \cdot U, \quad (49)$$

so that

$$\begin{cases} \cos \chi = g' \sqrt{\frac{g^2 + g'^2 + g''^2}{(g^2 + g'^2)(g'^2 + g''^2)}} = \frac{g' \hat{g}_Z}{\hat{g}' \hat{g}_Z}, \\ \sin \chi = g g'' / \sqrt{(g^2 + g'^2)(g'^2 + g''^2)} = \frac{g g''}{\hat{g}' \hat{g}_Z}, \\ \tan \chi = g g'' / g' \sqrt{g^2 + g'^2 + g''^2} = \frac{g g''}{g' \hat{g}_Z}, \end{cases} \quad (50)$$

giving back in particular (10,29) for $\epsilon = \tan \chi$.

B. The dark photon case

In the specific dark photon case for which $F = Y + F_d$, standard model particles do not interact with \hat{C} , only with \mathbf{W} and \hat{B} and thus \hat{Z} and \hat{A} . In the small m_U limit, \hat{A} in (46) appears for SM particles as a photon-like field, coupled to them only through their electromagnetic current, with strength

$$\hat{e} = g \sin \hat{\theta} = g \sqrt{\frac{g'^2 + g''^2}{g^2 + g'^2 + g''^2}} = \frac{g \hat{g}'}{\hat{g}_Z}. \quad (51)$$

The photon field, expressed as $A = \cos \chi \hat{A} - \sin \chi \hat{C}$ where \hat{C} is in this case uncoupled to SM particles, is coupled electromagnetically to them with the slightly re-

duced strength

$$\hat{e} \cos \chi = e = gg' / \sqrt{g^2 + g'^2}. \quad (52)$$

At the same time $U = \sin \chi \hat{A} + \cos \chi \hat{C}$ is coupled with the reduced strength $\hat{e} \sin \chi = e \tan \chi$. The A - \hat{A} angle χ , i.e. in fact the U - A mixing angle, is obtained from

$$\cos \chi = \frac{e}{\hat{e}} = g' \sqrt{\frac{g^2 + g'^2 + g''^2}{(g^2 + g'^2)(g'^2 + g''^2)}} = \frac{g' \hat{g}_Z}{\hat{g}' g_Z} = \frac{\sin \theta}{\sin \hat{\theta}}, \quad (53)$$

as found in (50) in the general case. U is here coupled to SM particles only through their electromagnetic current, with strength $\hat{e} \sin \chi = e \tan \chi$, i.e.

$$\epsilon e = e \tan \chi = g'' \frac{g^2}{g^2 + g'^2} \sqrt{\frac{g^2 + g'^2}{g^2 + g'^2 + g''^2}} = g'' \cos^2 \theta \cos \xi, \quad (54)$$

as directly found in (10) from the mixing of J_F^μ with $J_{Z_{\text{sm}}}^\mu$.

C. Recovering in a non-orthogonal basis the general expression of the U current

The $\hat{A}/A/U$ mixing in (46) provides an interpretation for the fact that the combination between the J_F^μ and $J_{Z_{\text{sm}}}^\mu$ neutral currents provides a U coupling to SM particles through the electromagnetic current. But is it possible to extend this interpretation to the general situation involving also B and L in the U couplings?

To do so, we express from (48) the U field in terms of the (non-orthogonal) A and \hat{C} fields defined in (4,46), as

$$U^\mu = \frac{\hat{C}^\mu}{\cos \chi} + \tan \chi A^\mu. \quad (55)$$

Its equation of motion reads

$$\partial_\mu U^{\mu\nu} = \mathcal{J}_U^\nu = \frac{1}{\cos \chi} \mathcal{J}_{\hat{C}}^\nu + \tan \chi \mathcal{J}_{\text{em}}^\nu. \quad (56)$$

With

$$\mathcal{J}_{\hat{C}}^\mu = -\sin \zeta \mathcal{J}_Y^\mu + \cos \zeta \mathcal{J}_F^\mu = \cos \zeta \frac{g''}{2} (\alpha J_B^\mu + \beta_i J_{L_i}^\mu + J_d^\mu), \quad (57)$$

the $U(1)$ gauge field \hat{C} is now coupled, not just to the dark matter current as in the pure “dark photon” case, but to a combination of it with the B and L currents.

Using the identity [42]

$$\frac{\cos \zeta}{\cos \chi} = \sqrt{\frac{g^2 + g'^2}{g^2 + g'^2 + g''^2}} = \cos \xi, \quad (58)$$

and $g'' \cos \xi \cos^2 \theta = e \tan \chi$, we recover from (55-57) expression (8) of \mathcal{J}_U^μ , or equivalently

$$\mathcal{J}_U^\mu = \epsilon e \left(J_{\text{em}}^\mu + \frac{1}{2 \cos^2 \theta} (\alpha J_B^\mu + \beta_i J_{L_i}^\mu + J_d^\mu) \right). \quad (59)$$

This also reads, in a grand-unified theory,

$$\mathcal{J}_U^\mu = \epsilon e \left(J_{\text{em}}^\mu - \frac{5}{4 \cos^2 \theta} J_{B-L}^\mu + \frac{1}{2 \cos^2 \theta} J_d^\mu \right), \quad (60)$$

in agreement with expression (33) of Q_U , with $\sin^2 \theta \simeq .238$ as appropriate for low energies.

The presence of the electromagnetic current, but also of the B and L currents, first found from the mixing (5,7) between J_F^μ and $J_{Z_{\text{sm}}}^\mu$ (cf. Fig. 1) [5], may be interpreted by expressing U^μ in terms of the non-orthogonal A^μ and \hat{C}^μ fields as in (55) (cf. Fig. 3). \mathcal{J}_U^μ is then obtained in terms of J_{em}^μ and J_d^μ , J_B^μ and $J_{L_i}^\mu$ as in (56-60). The specific case of a “dark photon” coupled proportionally to electric charges, simple to discuss and often used as a benchmark model, appears too restrictive, and the possible couplings of the U to the B and L currents should be taken into account as well.

D. “Kinetic mixing” as the effect of a description in a non-orthogonal field basis

As a side remark, the notion of “kinetic mixing”, popular now, has been used nowhere. It goes without saying that, as in *any* theory (including the standard model itself), using a non-orthogonal rather than an orthonormal field basis would *introduce* in the expression of the Lagrangian density non-diagonal terms, now often referred to as “kinetic-mixing” terms. These can always be eliminated by returning to an orthonormal basis, without any loss of physical content. Furthermore, and in contrast with a general belief, *it is not necessary that these “kinetic-mixing” terms be gauge invariant*, provided of course the complete Lagrangian density is invariant, independently of the basis in which it is expressed.

To illustrate this let us rewrite the Lagrangian density defined in terms of B^μ and C^μ as in (1-3), in the non-orthogonal basis (\hat{B}^μ, C^μ) , or (B^μ, \hat{C}^μ) , represented in Fig. 3, using from (44,46)

$$B^\mu = (\hat{B}^\mu / \cos \zeta) - \tan \zeta C^\mu. \quad (61)$$

We get

$$\begin{aligned} \mathcal{L}_k &= -\frac{1}{4} (W_3^{\mu\nu} W_{3\mu\nu} + B^{\mu\nu} B_{\mu\nu} + C^{\mu\nu} C_{\mu\nu}) \\ &= -\frac{1}{4} W_3^{\mu\nu} W_{3\mu\nu} \\ &\quad - \frac{1}{4} \frac{1}{\cos^2 \zeta} \left(\hat{B}^{\mu\nu} \hat{B}_{\mu\nu} + C^{\mu\nu} C_{\mu\nu} - 2 \sin \zeta \hat{B}^{\mu\nu} C_{\mu\nu} \right) \\ &= -\frac{1}{4} W_3^{\mu\nu} W_{3\mu\nu} \\ &\quad - \frac{1}{4} \frac{1}{\cos^2 \zeta} \left(B^{\mu\nu} B_{\mu\nu} + \hat{C}^{\mu\nu} \hat{C}_{\mu\nu} + 2 \sin \zeta B^{\mu\nu} \hat{C}_{\mu\nu} \right). \end{aligned} \quad (62)$$

This is immediately rediagonalized by returning to the orthogonal fields B^μ and C^μ (or \hat{B}^μ and \hat{C}^μ).

The same kinetic terms (62) in \mathcal{L} may be reexpressed in other non-orthogonal basis, involving different non-diagonal “kinetic-mixing” terms, this time not even gauge invariant. With

$$\begin{aligned} U^\mu &= \frac{\hat{C}^\mu}{\cos \chi} + \tan \chi A^\mu, \\ \text{or } A^\mu &= \frac{\hat{A}^\mu}{\cos \chi} - \tan \chi U^\mu, \end{aligned} \quad (63)$$

as in (55) one has

$$\begin{aligned} \mathcal{L}_k &= -\frac{1}{4} (Z^{\mu\nu} Z_{\mu\nu} + A^{\mu\nu} A_{\mu\nu} + U^{\mu\nu} U_{\mu\nu}) \\ &= -\frac{1}{4} Z^{\mu\nu} Z_{\mu\nu} \\ &\quad -\frac{1}{4} \frac{1}{\cos^2 \chi} (A^{\mu\nu} A_{\mu\nu} + \hat{C}^{\mu\nu} \hat{C}_{\mu\nu} + 2 \sin \chi A^{\mu\nu} \hat{C}_{\mu\nu}), \\ &= -\frac{1}{4} Z^{\mu\nu} Z_{\mu\nu} \\ &\quad -\frac{1}{4} \frac{1}{\cos^2 \chi} (\hat{A}^{\mu\nu} \hat{A}_{\mu\nu} + U^{\mu\nu} U_{\mu\nu} - 2 \sin \chi \hat{A}^{\mu\nu} U_{\mu\nu}), \end{aligned} \quad (64)$$

where the “kinetic-mixing” terms, which no longer involve abelian gauge fields only, are not gauge invariant.

The mixing angles between \hat{C}^μ and A^μ in (64), or \hat{C}^μ and B^μ in (62) are geometrically related by

$$\sin \chi = -\hat{C} \cdot A = \cos \theta \underbrace{(-\hat{C} \cdot B)}_{\sin \zeta} - \sin \theta \underbrace{\hat{C} \cdot W_3}_0, \quad (65)$$

thanks to the orthogonality between \hat{C} and W_3 (cf. Fig.3), i.e.

$$\sin \chi = \sin \zeta \cos \theta, \quad (66)$$

easily verified from $\sin \zeta \cos \theta = (g''/\hat{g}') (g/g_Z) = \sin \chi$.

Altogether there is no real gain in considering such non-diagonal kinetic terms, immediately re-diagonalized by returning to the original expressions. Considering kinetic-mixing terms i.e. using non-orthogonal field basis simply appears as a substitute for the introduction of the appropriate couplings of C^μ in the covariant derivative (1), involving both the visible and hidden sectors and leading to the corresponding current \mathcal{J}_F^μ in (3). This should not hide that C^μ may be coupled to B and L as well as to Y and dark matter, and the U boson to a combination of the electromagnetic with the B , L and dark matter currents.

In addition, a non-vanishing mixing angle χ , and coupling $\epsilon = e \tan \chi$ (also relating the visible and hidden sectors), corresponding to $U^\mu = (\hat{C}^\mu / \cos \chi) + \tan \chi A^\mu$ as in (55), may be obtained directly even in the presence of a single $U(1)$ gauge group, as in a $SU(5) \times U(1)_F$ gauge theory, with the visible and hidden sectors getting totally decoupled for $\epsilon = \chi = \zeta = \xi = 0$ [43]. These non-vanishing ϵ and χ are obtained here in spite of the non-existence of a gauge-invariant kinetic-mixing term between non-abelian ($SU(5)$) and abelian ($U(1)_F$) gauge fields.

VI. IMPLICATIONS FOR A LIGHT U

A. Axial couplings are strongly constrained

The axial couplings of the U should satisfy

$$(f_e^A f_q^V)^{\frac{1}{2}} \lesssim 10^{-7} m_U (\text{MeV}), \quad (67)$$

for m_U larger than a few MeVs, expressing that $|f_e^A f_q^V|/m_U^2 \lesssim 10^{-3} G_F$, to avoid too-large parity-violation effects in atomic physics [4]. A light U with axial couplings (1^+) could also be produced in a longitudinal polarization state with enhanced effective pseudoscalar couplings to quarks and leptons [1],

$$f_{ql}^P = f_{ql}^A \frac{2 m_{ql}}{m_U}, \quad (68)$$

much like a 0^+ pseudoscalar a . The resulting constraints on the axial couplings to heavy quarks, from ψ or $\Upsilon \rightarrow \gamma U$ and $K^+ \rightarrow \gamma U$ decays [3], are rather severe, especially for a light U with invisible decay modes into $\nu\bar{\nu}$ or LDM particles. In particular the axial couplings to down quarks and charged leptons, universal when they get masses from the same doublet v.e.v. (as in supersymmetric theories) must then verify [9, 28, 29]

$$f_{e,d}^A \lesssim (2 \text{ to } 4) 10^{-7} m_U (\text{MeV}) / \sqrt{B_{\text{inv}}} \quad (69)$$

(which is typically $\lesssim 10^{-5}$ for a U in the ~ 10 MeV mass range). This leads, in such cases where axial couplings may occur, to consider a $U(1)_F$ symmetry broken at a scale larger than electroweak through a large singlet v.e.v., very much as for an “invisible” axion [2].

The U lifetime may vary considerably between less than 10^{-15} s to many years and even infinity, depending on its mass and couplings. The decay rate for $U \rightarrow e^+ e^-$ is given by [1]

$$\begin{aligned} \Gamma_{ee} &\simeq \frac{1}{12\pi} \left[(f_e^V)^2 \left(1 + \frac{2m_e^2}{m_U^2} \right) \sqrt{1 - \frac{4m_e^2}{m_U^2}} \right. \\ &\quad \left. + (f_e^A)^2 \left(1 - \frac{4m_e^2}{m_U^2} \right)^{3/2} \right] m_U, \end{aligned} \quad (70)$$

including the phase-space factors $\frac{3}{2}\beta - \frac{1}{2}\beta^3$ and β^3 for the vector and axial production of spin- $\frac{1}{2}$ particles, with $\beta = \sqrt{1 - 4m_e^2/m_U^2}$. We also have to take into account the invisible decay modes of the U into ordinary neutrinos, given for three left-handed neutrinos with chiral couplings f_ν by

$$\Gamma_{\nu_L \bar{\nu}_L} \simeq \frac{f_\nu^2}{8\pi} m_U, \quad (71)$$

and other possible invisible decays into right-handed neutrinos and light dark matter particles, which could decrease significantly the branching ratio for $U \rightarrow e^+ e^-$.

B. Vector couplings should not be too large

Let us thus return to a light U vectorially coupled to SM particles, as is the case when a single doublet φ contributes to the electroweak breaking (or several but with the same gauge quantum numbers) [5]. Couplings proportional to a combination of B , L_i and Q as in (9,40,60), rather than just Q , open new possibilities for experimental detection [28]. Experimental results, usually discussed in the $(\log \epsilon, m_U)$ plane, should also be considered in terms of these couplings $f = \epsilon e Q_U$, through the changes in the couplings to SM particles

$$\epsilon e Q \rightarrow f = \epsilon e Q_U = \epsilon e (Q + \lambda_B B + \lambda_{L_i} L_i), \quad (72)$$

leading to $f_n + f_{\nu_e} = f_p + f_e$ as in (13), and more specifically

$$\epsilon e Q \rightarrow \epsilon e Q_U = \epsilon e (Q - \lambda(B - L)), \quad (73)$$

with $Q_U(e) = \lambda - 1 \simeq .64$ in a grand-unified theory. This usually results in moderate shifts of the various limits when expressed in terms of $\log \epsilon$; e.g. for an experiment sensitive to the couplings to the electron, through the change

$$(\epsilon e)^2_{\text{dark photon}} \rightarrow [f_e = \epsilon e Q_U(e)]^2 \times B_{ee}. \quad (74)$$

The U boson should interact sufficiently weakly with electrons, so that its contribution to the electron anomaly,

$$\delta U a_e \simeq \frac{f_e^2}{12\pi^2} \frac{m_e^2}{m_U^2} \simeq \frac{\alpha}{3\pi} [\epsilon Q_U(e)]^2 \frac{m_e^2}{m_U^2} \quad (75)$$

for m_U larger than a few MeVs, be less than about $3 \cdot 10^{-12}$ [30] (improved over the earlier $2 \cdot 10^{-11}$ leading to $|f_e| \lesssim 10^{-4} m_U(\text{MeV})$ [9]). This requires

$$|f_e = \epsilon e Q_U(e)| \lesssim 4 \cdot 10^{-5} m_U(\text{MeV}), \quad (76)$$

or

$$|\epsilon Q_U(e)| \lesssim 1.2 \cdot 10^{-3} \frac{m_U}{10 \text{ MeV}}, \quad (77)$$

applicable for m_U above a few MeVs.

The U should also act sufficiently weakly with protons, so that the $\pi^0 \rightarrow \gamma U$ decay rate be sufficiently small. The corresponding branching ratio is given, for m_U somewhat below m_{π^0} , by $2(2f_u + f_d)^2/e^2 = 2\epsilon^2 Q_U(p)^2$, replacing the $2\epsilon^2$ of the pure dark photon case, typically constrained to be $\lesssim 10^{-6}$ for m_U in the 10-100 MeV range [31]. This experiment provides similar limits for a U boson decaying into e^+e^- , with the replacement

$$(\epsilon e)^2_{\text{dark photon}} \rightarrow \underbrace{[f_p = 2f_u + f_d]}_{\epsilon e Q_U(p)}^2 \times B_{ee}, \quad (78)$$

leading to

$$|f_p = \epsilon e Q_U(p)| \lesssim 3 \cdot 10^{-4} / \sqrt{B_{ee}}, \quad (79)$$

with $Q_U(p) = 1 - \lambda \simeq -.64$ in a grand-unified theory.

For a boson with significant invisible decays into neutrinos or light dark matter particles, we have, from the search for the decay $\pi^0 \rightarrow \gamma + U_{\text{inv.}}$ [32–34], with a branching ratio fixed by $2\epsilon^2 Q_U(p)^2 < 3.3 \cdot 10^{-5}$ for $m_U < 120 \text{ MeV}$, the limit

$$|f_p = \epsilon e Q_U(p)| \lesssim 1.2 \cdot 10^{-3} / \sqrt{B_{\text{inv.}}} \quad (80)$$

We also have, from a low- q^2 ν_e - e scattering experiment [35], the constraint $|f_{\nu_e} f_e|/m_U^2 \lesssim G_F$ [4, 7], i.e.

$$|f_{\nu_e} f_e|^{1/2} \lesssim 3 \cdot 10^{-6} m_U(\text{MeV}), \quad (81)$$

valid for m_U larger than a few MeVs (or $|f_{\nu_e} f_e|/m_U^2 \lesssim 10^{-5}$ otherwise), also expressed as

$$\epsilon |Q_U(\nu_e) Q_U(e)|^{1/2} \lesssim 10^{-4} \frac{m_U}{10 \text{ MeV}}. \quad (82)$$

If f_e, f_p and f_{ν_e} are all small as suggested by the above constraints, f_n may have to be small as well, as a result of (13).

For an anomaly-free theory the currents may be constructed from Q , $B - L$ (with ν_R 's) and $L_i - L_j$, with the dark matter current, also vectorial, involving spin-0 or Dirac spin-1/2 dark matter particles. With a family-independent symmetry Q_U involves $B - L$ as in (15), $Q_U = Q - \lambda(B - L) + Q_{U\text{dark}}$, as found in a grand-unified theory, implying

$$f_e = -f_p = (\lambda - 1)\epsilon e, \quad f_{\nu_e} = -f_n = \lambda\epsilon e, \quad (83)$$

in agreement with (13). More specifically the constraint (81,82) reads $\epsilon e \sqrt{|\lambda(\lambda - 1)|} \lesssim 3 \cdot 10^{-5} m_U/(10 \text{ MeV})$ and typically implies, in a grand-unified theory with $\lambda \simeq 1.64$ so that $\sqrt{|\lambda(\lambda - 1)|} \simeq 1$,

$$\epsilon \lesssim 10^{-4} \frac{m_U}{10 \text{ MeV}}, \quad (84)$$

for m_U larger than a few MeVs.

Still it may be possible, although at the price of elegance, to arrange for ν_e , or ν_e and ν_μ , not to interact with U , e.g. through the change $B - L \rightarrow B - 3L_\tau$, leading to

$$Q_U = Q - \lambda(B - 3L_\tau) + Q_{U\text{dark}}, \quad (85)$$

still in an anomaly-free theory. More specifically

$$Q_U \simeq Q - (B - 3L_\tau) + Q_{U\text{dark}} \quad (86)$$

would lead to $Q_U(n) \simeq Q_U(e) \simeq Q_U(\mu) \simeq -1$, $Q_U(p) \simeq Q_U(\nu_e) \simeq Q_U(\nu_\mu) \simeq 0$, i.e.

$$f_n \simeq f_e \simeq -\epsilon e, \quad \text{with } f_p, f_{\nu_e}, f_{\nu_\mu} \text{ very small.} \quad (87)$$

The U couplings in (9) offer new opportunities for tentative interpretations of possible anomalies. The decays

of excited states of ^8Be have long been viewed as potentially sensitive to the production of light spin-1 U bosons and anomalous production of e^+e^- pairs that could signal such decays have already been reported [36], although these first indications were not confirmed. A possible new anomaly has been found recently [37], which remains to be better understood before attempting at an interpretation [38, 39]. It does not seem that it can be attributed to a dark photon coupled proportionally to electric charges, which would require a too large ϵ . Expressions (8,9) of the U charge and current [5] may help provide an interpretation if the effect is real, possibly with a U interacting more strongly with neutrons and electrons than with protons and neutrinos, keeping in mind relations (11-13) associated with a conserved Q_U in the massless limit.

VII. CONCLUSIONS

Expression of Q_U from Q , B and L

The structure presented here for the interactions of a light U boson depends on a very small number of relevant parameters, especially ϵ and m_U with Q_U expressed as $Q + \lambda_B B + \lambda_i L_i [+Q_d]$ or $Q - \lambda(B - L) [+Q_d]$. It provides a consistent framework to deal with a new interaction, naturally parity-conserving in the visible sector and coupled to a conserved charge Q_U , in the small mass limit. The U may be viewed as *mixed with the Z* , or *mixed with the photon*, or both at the same time with a 3×3 mixing matrix extending the 2×2 electroweak one of the standard model.

Kinetic mixing as an effect of a non-orthogonal field basis

What is often referred to as “kinetic mixing” simply corresponds to choosing a description in a non-orthogonal field basis. This also implies that the kinetic-mixing terms associated with this basis are not even required to be gauge invariant, in contrast with a common belief. Furthermore, even with the U viewed as kinematically mixed with the photon, B and L contributions are generally allowed in its couplings, and may even be required as in the case of grand-unification.

A non-vanishing $\epsilon = \tan \chi$ within grand-unification

The construction is compatible with grand-unification, with a charge Q_U commuting with the electrostrong symmetry between the photon and gluons at the GUT scale, and a *non-vanishing $\epsilon = \tan \chi$ already present at the classical level*. This occurs in spite of the fact that the $SU(5) \times U(1)_F$ gauge group includes a *single abelian factor* $U(1)_F$, so that no gauge-invariant kinetic term mixing the $SU(5)$ and $U(1)_F$ gauge fields may be written. Still it is possible to view the U as (“kinematically”) mixed with the photon as in (55,63), with non gauge-invariant mixing terms in the lagrangian density as in (64), and a U boson also coupled to the B and L currents.

Q_U from the $SU(4)_{\text{es}} \times U(1)_U$ electrostrong symmetry

Q_U evolves, from $Q - 2(B - L) + Q_{U \text{ dark}}$ at the GUT scale, to

$$Q_U \simeq Q - 1.64(B - L) + Q_{U \text{ dark}} \quad (88)$$

at low energy. This expression, and the more general one $Q_U \simeq Q - \lambda(B - L) + Q_{U \text{ dark}}$, motivated by grand-unification and by anomaly-cancellation, may be used to *display the experimental constraints in the $(\log \epsilon, m_U)$ plane as a function of λ* .

The U current in the visible sector is purely vectorial in the massless U limit, in relation with the fact that the theory may admit at high energy *an extended $U(4) = SU(4)_{\text{es}} \times U(1)_U$ electrostrong symmetry which preserves parity*, with the U , photon, gluons and $X^{\pm 4/3}$ bosons all coupled to vector currents. The interactions mediated by the Z and the electrostrong quartet ($Y^{\mp 1/3}, W^{\pm}$), on the other hand, violate parity.

* * *

A large variety of interesting effects may occur, in particular for a U in the $\approx \text{MeV}$ to hundred MeVs mass range. The U boson, if extremely light or massless, may also lead to a new long range force, extremely weak, that could manifest through apparent violations of the Equivalence Principle. This one will soon be tested in space to an increased level of precision. The characteristics of the new interaction mediated by such a light neutral spin-1 U boson may also play a role in shedding light on a possible unification of weak, electromagnetic and strong interactions.

-
- [1] P. Fayet, Nucl. Phys. B **187**, 184 (1981), “In search of a new spin-1 boson”
 - [2] P. Fayet, Phys. Lett. B **95**, 285 (1980), “Effects of the spin-1 partner of the goldstino (gravitino) on neutral current phenomenology”
 - [3] P. Fayet, Phys. Lett. B **172**, 363 (1986), “The fifth interaction in grand-unified theories: a new force acting mostly on neutrons and particle spins”

- [4] C. Bouchiat and P. Fayet, Phys. Lett. B **608**, 87 (2005), “Constraints on the parity-violating couplings of a new gauge boson”
- [5] P. Fayet, Phys. Lett. B **227**, 127 (1989), “The fifth force charge as a linear combination of B , L (or $B - L$) and electric charges”; Nucl. Phys. B **347**, 743 (1990), “Extra $U(1)$ ’s and new forces”
- [6] P. Fayet, Class. Quant. Grav. **13**, A19 (1996), “New in-

- interactions and the Standard Models”
- [7] C. Boehm and P. Fayet, Nucl. Phys. B **683**, 219 (2004), “Scalar dark matter candidates”
 - [8] P. Fayet, Phys. Rev. D **70**, 023514 (2004), “Light spin- $\frac{1}{2}$ or spin-0 dark matter particles”
 - [9] P. Fayet, Phys. Rev. D **75**, 115017 (2007), “ U -boson production in e^+e^- annihilations, ψ and Υ decays, and light dark matter”
 - [10] M. Pospelov, Phys. Rev. D **80**, 095002 (2009), “Secluded $U(1)$ below the weak scale”
 - [11] A. Hoecker and W. Marciano, Chin. Phys. C **38**, 090001 (2014), “The muon anomalous magnetic moment”
 - [12] T.D. Lee and C.-N. Yang, Phys. Rev. **98**, 1501 (1955) “Conservation of heavy particles and generalized gauge transformations”
 - [13] E. Adelberger *et al.*, Phys. Rev. Lett. **59**, 849 (1987), “New constraints on composition-dependent interactions weaker than gravity”
 - [14] B. Heckel *et al.*, Phys. Rev. Lett. **63**, 2705 (1989), “Experimental bounds on interactions mediated by ultralow mass bosons”
 - [15] S. Schlamminger *et al.*, Phys. Rev. Lett. **100**, 041101 (2008), “Test of the equivalence principle using a rotating torsion balance”
 - [16] T. Wagner *et al.*, Class. Quant. Grav. **29**, 1184002 (2012), “Torsion-balance tests of the weak equivalence principle”
 - [17] J. Bergé, P. Touboul and M. Rodrigues, J. Phys., Conf. Ser. **610**, 012009 (2015), <https://arxiv.org/abs/1501.01644>, “Status of MICROSCOPE, a mission to test the Equivalence Principle in space”
 - [18] T. Damour, Class. Quant. Grav. **29**, 184001 (2012), “Theoretical aspects of the Equivalence Principle”
 - [19] ATLAS coll.: Phys. Lett. B **716**, 1 (2012), “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC”
 - [20] CMS coll.: Phys. Lett. B **716**, 30 (2012), “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC”
 - [21] P. Galison and A. Manohar, Phys. Lett. B **136**, 279 (1984), “Two Z ’s or not two Z ’s?”
 - [22] B. Holdom, Phys. Lett. B **166**, 196 (1986), “Two $U(1)$ ’s and ϵ charge shifts”
 - [23] F. Bossi *et al.*, “Dark forces at accelerators” (2012), <https://agenda.infn.it/conference0therViews.py?view=standard&confId=4897>
 - [24] J. Alexander *et al.*, <https://arxiv.org/abs/1608.08632>, “Dark Sectors 2016 Workshop”, and references therein
 - [25] H. Georgi and S. Glashow, Phys. Rev. Lett. **32**, 438 (1974), “Unity of all elementary particle forces”
 - [26] H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974), “Hierarchy of Interactions in unified gauge theories”
 - [27] K. Abe *et al.*, the Super-Kamiokande Collaboration, <https://arxiv.org/abs/1610.03597>, “Search for proton decay via $p \rightarrow e^+\pi^0$ and $p \rightarrow \mu^+\pi^0$ in the Super-Kamiokande water Cherenkov detector”
 - [28] P. Fayet, Phys. Rev. D **74**, 054034 (2006), “Constraints on light dark matter and U bosons, from ψ , Υ , K^+ , π^0 , η and η' decays”
 - [29] P. Fayet, Phys. Lett. B **675**, 267 (2009), “ $U(1)_A$ symmetry in two-doublet models, U bosons or light scalars, and ψ and Υ decays”
 - [30] T. Aoyama *et al.*, Phys. Rev. Lett. **109**, 111807 (2012), “Tenth-order QED contribution to the electron $g-2$ and an improved value of the fine structure constant”
 - [31] NA48/2 Coll., J. Batley *et al.*, Phys. Lett. B **746**, 178 (2015), “Search for the dark photon in π^0 decays”.
 - [32] M. Atiya *et al.*, Phys. Rev. Lett. **69**, 733 (1992), “Search for the decay $\pi^0 \rightarrow \gamma + X$ ”
 - [33] C. Amsler *et al.*, Crystal Barrel Coll., Phys. Lett. B **333**, 271 (1994), “Search for a new light gauge boson in decays of π^0 and η ”
 - [34] J. Altegoer *et al.*, NOMAD Coll., Phys. Lett. B **428**, 197 (1998), “Search for a new gauge boson in π^0 decays”
 - [35] L. Auerbach *et al.*, LSND Coll., Phys. Rev. D **63**, 112001 (2001), “Measurement of $\nu_e e$ elastic scattering”
 - [36] A. Vitez *et al.*, Acta Phys. Polon. B **39**, 483 (2008), <https://inspirehep.net/record/807795/files/v39p0483>, “Anomalous internal pair creation in ^8Be as a signature of the decay of a new particle”
 - [37] A. Krasznahorkay *et al.*, Phys. Rev. Lett. **116**, 042501 (2016), “Observation of anomalous internal pair creation in ^8Be : A possible indication of a light, neutral boson”
 - [38] J. Feng *et al.*, Phys. Rev. Lett. **117**, 071803 (2016), “Protophobic fifth-force interpretation of the observed anomaly in ^8Be nuclear transitions”
 - [39] J. Feng *et al.*, <https://arxiv.org/abs/1608.03591>, “Particle Physics Models for the 17 MeV Anomaly in Beryllium Nuclear Decays”
 - [40] Two doublets, such as h_1 and h_2^c in supersymmetric theories, both with $Y = -1$, also allow for a $U(1)_A$ symmetry rotating differentially the two doublets and acting axially on quarks and leptons. Its generator F_A may provide an extra contribution to F and \mathcal{J}_F^μ in (1-3), the resulting current \mathcal{J}_U^μ including an axial part. A light U boson is then produced very much like a light pseudoscalar a , with the $U(1)_F$ symmetry broken at a scale larger than electroweak through a large singlet v.e.v.. This ensures that this effective pseudoscalar a has reduced interactions, very much as for an invisible axion [1, 2].
 - [41] Even if the notations suggest that C^μ is a free field for $g'' = 0$, we can return to (2,3) to still view as C^μ as possibly interacting with the hidden sector through the dark current \mathcal{J}_d^μ .
 - [42] This may be seen geometrically from $\cos \zeta = C \cdot (\hat{C} = -\sin \chi A + \cos \chi U) = \cos \chi (C \cdot U) = \cos \chi \cos \xi$, and verified using the expressions of $\cos \xi$, $\cos \zeta$ and $\cos \chi$ in (6,44,50).
 - [43] There is thus no need to invoke hypothetical effects of radiative corrections for generating a non-vanishing value of ϵ , especially when no coupling between the visible and hidden sectors is present.